

Question 7.3–1: (Solution, p 3) Suppose we have a perceptron with three inputs, and the perceptron's current weights are $\langle 0.5, 0.2, -0.5 \rangle$.

- a. What would the perceptron predict given the input $\langle 1, -1, -1 \rangle$?
- b. Suppose this prediction were wrong. How would the perceptron update its weights if the learning rate were $r = 0.1$?

Question 7.3–2: (Solution, p 3) Suppose we have a three-input perceptron, where input x_1 is permanently wired to a 1 input.

- a. Suppose the weights of the perceptron were $\langle 0.1, 0.5, -0.4 \rangle$, and we were to give it the inputs $\langle 1, 1, 1 \rangle$. What would the perceptron output?
- b. Suppose we wanted to select weights so that our perceptron would behave like the AND of the inputs x_2 and x_3 .

x_1	x_2	x_3	o
1	-1	-1	-1
1	-1	1	-1
1	1	-1	-1
1	1	1	1

Give a combination of perceptron weights w_1, w_2 , and w_3 that would result in the desired behavior.

Question 8.1–1: (Solution, p 3) Consider the following context-free grammar.

$$\mathbf{S} \rightarrow \varepsilon \mid a \mathbf{S} b \mathbf{S} \mid b \mathbf{S} a \mathbf{S}$$

Give a derivation of the string *aabbab* using this grammar.

Question 8.1–2: (Solution, p 3) Consider the following context-free grammar.

$$\begin{aligned} \mathbf{S} &\rightarrow \mathbf{V} \mathbf{V} \mid \mathbf{V} \\ \mathbf{V} &\rightarrow (\mathbf{S}) \mid x \mid y \mid z \end{aligned}$$

Each of the following sentences is either described by this grammar or not. If it is, give either a parse tree with **S** at its root or a derivation from **S**. If it is not, simply say so.

- a. $x(y)$
- b. xyz
- c. $x(yz)$

Question 8.1–3: (Solution, p 3) Write a context-free grammar describing each of the following languages.

- a. the set of strings of *a*'s and *b*'s beginning and ending with the letter *a*
- b. the set of strings containing either only *a*'s or only *b*'s
- c. the set of strings of left-brackets and right-brackets where the brackets match (you can pair all brackets so that each pair includes a left bracket and a right bracket occurring after it in the string). Examples include $[[[]]]$ and $[[[[[]]]]]$.

Question 8.2–1: (Solution, p 4) Write a regular expression describing each of the following languages.

- a. strings containing only a 's and b 's where all a 's come before all b 's.
- b. strings containing only a 's and b 's in which “ aab ” somewhere occurs as an adjacent sequence (like $aabaa$ or $baaba$ but not $abbab$).
- c. binary representations of positive even numbers.

Question 8.2–2: (Solution, p 4) Give a regular expression for each of the following languages.

- a. all strings containing an even number of a 's (and no other letters).
- b. all strings that contain either only a 's or only b 's.
- c. all representations of binary numbers that are at least 4.

Question 8.2–3: (Solution, p 4) Give an English description of a language that can be described by a context-free grammar but not by a regular expression.

Solution 7.3–1: (Question, p 1)

- a. It would predict 1.
- b. $\langle 0.4, 0.3, -0.4 \rangle$

Solution 7.3–2: (Question, p 1)

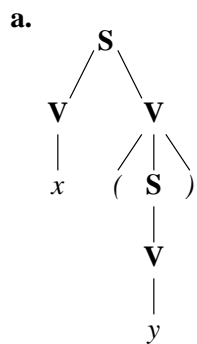
- a. 1 (The weighted sum is 0.2, which exceeds 0.)
- b. $\langle -0.5, 1, 1 \rangle$ (There are many other correct answers.)

Solution 8.1–1: (Question, p 1)

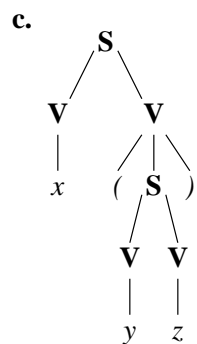
$S \Rightarrow aSbS$
 $\Rightarrow aaSbSbS$
 $\Rightarrow abSbS$
 $\Rightarrow aabbSaSbS$
 $\Rightarrow aabbaSbS$
 $\Rightarrow aabbabS$
 $\Rightarrow aabbab$

[Note that it is important in a derivation to replace exactly one symbol in each step. Do not combine steps.]

Solution 8.1–2: (Question, p 1)



- b. The sequence is not described by the grammar



Solution 8.1–3: (Question, p 1)

a.

$$\begin{aligned} \mathbf{S} &\rightarrow a \mathbf{T} a \\ \mathbf{T} &\rightarrow a \mathbf{T} \mid b \mathbf{T} \mid \varepsilon \end{aligned}$$

b.

$$\begin{aligned} \mathbf{S} &\rightarrow \mathbf{A} \mid \mathbf{B} \\ \mathbf{A} &\rightarrow a \mathbf{A} \mid \varepsilon \\ \mathbf{B} &\rightarrow b \mathbf{B} \mid \varepsilon \end{aligned}$$

c.

$$\mathbf{S} \rightarrow \mathbf{S} \mathbf{S} \mid [\mathbf{S}] \mid \varepsilon$$

Solution 8.2–1: (Question, p 2)a. a^*b^* b. $(a \mid b)^*aab(a \mid b)^*$ c. $1(0 \mid 1)^*0$ **Solution 8.2–2:** (Question, p 2)a. $(aa)^*$ b. $a^* \mid b^*$

c. $1(0 \mid 1)(0 \mid 1)(0 \mid 1)^*$ [Binary numbers that are at least 4 must have at least three bits. This regular expression describes all combinations of 0's and 1's that begin with a 1, have two more bits, and can have any number of bits thereafter.]

Solution 8.2–3: (Question, p 2) The language of palindromes containing a 's and b 's is a context-free language, but it is not a regular language.