Assignment 2, Math 240, Fall 2005

Due: September 6. Value: 30 pts.

Based on August 30 material

- §1.5 (p 73): 4
- If x^2 is irrational, then x is irrational.

  a. Prove it.

  b. What kind of proof did you use for the previous problem (direct, indirect, vacuous, trivial, contradiction)?
- Is the product of two irrational numbers always rational, always irrational, or sometimes rational and sometimes not? Prove your answer.
- Suppose n is a three-digit number — i.e., $n = 100a + 10b + c$ for three integers a, b, and c between 0 and 9. Prove that n is a multiple of 3 if and only if the sum of n's digits (i.e., $a + b + c$) is a multiple of 3.

Based on September 1 material

- §2.4 (p 167): 14. The answer is 24, but argue that this is the answer without resorting to a calculator or computer.
- §2.4 (p 167): 26 (ϕ is defined in 25)
- §2.4 (p 168): 46. This is implied if you can show that any divisor of both a and m also is also a divisor of both b and m. (You don't need to mention this, but the symmetric argument will also show that any divisor of b and m is also a divisor of a and m. As a result, the two sets of divisors are the same, and so the greatest in each set is the same.)
- The previous problem leads to the **Euclidean algorithm** for computing greatest common divisors: The GCD of a and 0 is a, while the GCD for a and a non-zero b can be computed by computing the GCD of $a \bmod b$ and b.

  a. Use this algorithm to find the following GCDs, showing your intermediate steps:

  i. $gcd(40, 30)$

  ii. $gcd(84, 60)$

  b. Explain why the Euclidean algorihm is related to the previous problem.