Assignment 5, Math 240, Fall 2005

Due: 2:45pm, September 27. Value: 25 pts.

Based on September 20 material

Problem A. §3.4 (p 271), 12.

Problem B. For each of the below functions, evaluate $f(2)$, $f(3)$, $f(4)$, and $f(5)$.

a. $f(0) = 1$
for $n \geq 1$, $f(n) = f(n-1) + f\left(\left\lfloor \frac{n}{2} \right\rfloor\right)$

b. $f(-1) = 2$
for $n \geq 0$, $f(n) = n + f(n-1) + 1$

Problem C. Suppose I tell you I am thinking of a set A, and my set has three properties.

A is a set of integers (i.e., $A \subseteq \mathbb{Z}$)
$2 \in A$
for any two integers a and b from A, $a + b$ is also in A

a. Describe at least four sets that I may be thinking of.

b. Suppose we define S to be the intersection of all such possibilities: That is, an element is in S if it is in all sets that satisfy the properties specified for A.

Give a simpler characterization of S: That is, what particular elements are in it? Justify your answer.

Problem D. Prove, using strong induction, that every positive integer is the sum of *distinct* powers of 2. (For example, 22 is the sum of 2^4, 2^2, and 2^1, which are all different powers of 2. Note that you do not need to prove that there is only one such set (though there is).)

Based on September 22 material

Problem E. §3.4 (p 272): 39.

Problem F. §3.4 (p 272): 38. A *palindrome* is a string that reads the same backwards and forwards; examples include *redder* and *dewed*, although in this case your strings will consist of 0's and 1's.

Prove using structural induction that all strings generated by your definition are palindromes. (You do not need to prove that all palindromes are generated by your definition, but this should be true also.)

Problem G. §3.4 (p 271): 26ac.