Take-Home Test 2, Math 240, Fall 2005

Due: 2:45pm, October 4. Value: 50 pts.

Instructions: In contrast to assignments, you must not discuss this take-home test with others, and you may not use electronic resources. The only written resources you may use are your textbook and class notes that you have written yourself. Be careful not to place your test where others might read your solutions. You may discuss the take-home test with me, although I won't promise to be helpful.

I will schedule a time for October 5 or 6 when you will formally present at least two of your solutions to me. A portion of your grade will be based on your presentation. (If you are late for your appointment, or if you miss it, you will receive a penalty.) I recommend keeping a copy of your work so that you can study your solutions before presenting them; your presentation, though, should not involve pointing to your copy.

You will have the opportunity to complete a "rewrite," due the class day following when I distribute your initial grades. You will receive a three-point penalty on the rewrite, and your overall grade will be the higher of your initial grade and your rewrite grade. Your first graded version must be submitted with your rewrite.

Because of the rewrite policy, you should be careful not to discuss the test with others or access related resources even *after* you have submitted your initial version.

Problem A. Recall that we defined f_n is defined to be the nth Fibonacci number, starting with f_0 as 0 and f_1 as 1. Suppose that we define a new sequence g_n defined where

$$g_n = \begin{cases} 0 & \text{if } n = 0 \\ g_{n-1} + f_n & \text{if } n > 0 \end{cases}$$

Compute the first nine values $g_1, g_2, \ldots g_9$, and form a hypothesis relating the g_n sequence to a sequence you already know.

Prove (using induction) that your hypothesis is correct.

Problem B. Suppose we have a bag filled with three types of Legos: 1×1 white pieces, 2×1 blue pieces, and 2×1 red pieces. Somebody has given us a $n \times 1$ green piece, and we want to place white, blue, and red pieces to cover the top of it. For example, if our green piece were 3×1, then we would have five possible ways to accomplish the goal: WWW, WR, WB, BW, RW.

Prove using strong induction that for every n we have

$$\frac{2^{n+1} + (-1)^n}{3}$$

ways of accomplishing our goal.

Problem C. Consider the language L defined by the following rules.

$\lambda \in L$
if $w \in L$, then $0w0 \in L$
if $w \in L$ and $x \in L$, then $w1x \in L$

Using structural induction, prove that all strings in L have an even number of 0's. As always, be explicit about when you use the induction hypothesis.