Math 240, Fall 2005, Exam 1

Name: ____________________________

1. [8 pts]

a. Relative to the implication $p \rightarrow q$, what name is given to $\neg q \rightarrow \neg p$?

b. Complete the below truth table for the expression $\neg q \rightarrow \neg p$.

p	q	(optional intermediate work)	$\neg q \rightarrow \neg p$
F	F		
F	T		
T	F		
T	T		

2. [8 pts] Translate the following statements into logical expressions using the following symbols.

$K(x)$	x is wearing socks.	$C(x)$	x is in this class.
$S(x)$	x is wearing shoes.	$Burch$	Dr. Carl Burch

a. Dr. Burch must be wearing socks to wear his shoes.

b. Somebody in this class is wearing socks but not shoes.

c. At least two different people in this class are wearing shoes.

3. [8 pts] Show $\neg s$ using a formal proof based on the given hypotheses. Listed at right are some rules of inference that might possibly prove useful.

1.	$r \wedge s \rightarrow t$	hypothesis
2.	$\neg t$	hypothesis
3.	r	hypothesis

$$\frac{p \wedge q}{p} \quad \text{Simplification}$$

$$\frac{\begin{array}{l}\neg q \\ p \rightarrow q\end{array}}{\neg p} \quad \text{Modus tollens}$$

$$\frac{\begin{array}{l}p \vee q \\ \neg p\end{array}}{q} \quad \text{Disjunctive syllogism}$$

4. [8 pts] Give an integer value for n **between 10 and 23** for which each of the following is true. (You can use different numbers for different expressions.)

a. $3 \equiv n \pmod 6$

b. $9 \mid n$

c. $25 \bmod n = 1$

d. $gcd(24, n) = 4$

5. [8 pts] In Assignment 3, we saw the **Euclidean algorithm** for computing greatest common divisors: The GCD of a and 0 is a, while the GCD for a and a non-zero b can be computed by computing the GCD of $a \bmod b$ and b.

Use this algorithm to compute $gcd(84, 60)$, showing your intermediate steps.

6. [10 pts] Prove that for all integer values of a, c, and d, $d \mid a$ implies $dc \mid ac$.

7. [10 pts] Prove that every positive composite integer n has a factor that is more than 1 but not more than $\sqrt{n}$.

8. [10 pts] Consider the function $f : \mathbb{N} \to \mathbb{N}$ defined as $f(x) = \lceil \sqrt{2x} \rceil$.

a. Is f one-to-one? Explain.

b. Is f onto? Explain.

9. [15 pts] We saw in class that the set of positive rational numbers is countable. Describe a one-to-one, onto function (possibly the same one we saw in class) from $\mathbb{N}$ to the positive rationals.

Supposing this function were named f, compute the specific rational number to which each of the following maps.

i. $f(0) \quad =$

ii. $f(4) \quad =$

iii. $f(7) \quad =$

iv. $f(10) \quad =$

10. [15 pts] Prove using induction that the sum of the first n odd numbers, i.e.

$$\sum_{i=1}^{n}(2i-1)\,,$$

is n^2. Your basis step should be for $n = 1$.