Assignment 4, Math 240, Fall 2005

Due: 2:45pm, September 15. Value: 14 pts.

(Note that the first exam is Thursday, September 15.)

Based on September 13 material

Proof by induction is discussed in the textbook in §3.3.

Problem 1. Prove using induction that for every integer $n \geq 1$, $2^{3n} - 1$ is divisible by 7.

Problem 2. The following "proof" is obviously wrong, because the conclusion it presents is ridiculous. Explain the specific point where the proof goes wrong — that is, you should be able to point to a specific place in the proof where it does something invalid.

> **Theorem:** For every integer $n \geq 0$, all subsets of the natural numbers have at least n elements.
>
> **Proof:** Basis step ($n = 0$): Sets cannot have a negative size, so the size must be at least 0.
>
> Inductive step: Suppose it were true that all sets have at least n elements; we want to show that all sets must then have at least $n + 1$ elements. Consider, then, any set S. Choose some element x from S, and define S' to be $S - \{x\}$. Because S' is a set, it must have at least n elements according to our inductive hypothesis. Adding x into S' gives us S again, and it must have $n + 1$ elements, because S' has n elements and $x \notin S'$. We've shown, then, that every set must have at least $n + 1$ elements.

Problem 3. Consider a game played with a pile of candies, with turns alternating between two players. Each turn, a player can eat one to three candies. The winner is the player who eats the last candy. Prove that if the initial number of candies is divisible by four, then the first player has a strategy to win. (Hint: Use induction on n, where the initial number of candies is $4n$.)