Assignment 7, Math 240, Fall 2005

Due: September 29. Value: 5 pts.

Problem A. In class we defined the **height** of a tree using a one-node tree as our basis (with a height of 0); we extend this definition to include empty trees by defining the height of an empty tree to be -1.

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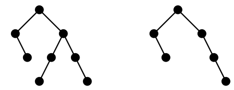
An **AVL tree** is one in which for each vertex, its subtrees' heights differ by at most one. For example, consider the two trees below.



AVL tree Not AVL

You can confirm that the tree at left is an AVL tree by looking at each node and comparing the heights of its two subtrees: In the case of the root, the two subtrees' heights are 1 and 2, which differ by 1; for the root's right child, the two subtrees' heights are 1 and 1, which differ by 0; and so on. The tree at right is not at AVL tree because the root's right child has subtree heights of -1 and 1.

We can also talk about the *smallest* AVL tree for a particular height. In fact, if we remove one of the bottom nodes in the AVL tree above, then we would have the smallest AVL tree of height 3. (You can verify that removing any additional nodes would disqualify it either from being an AVL tree or from having height 3.)

Prove using structural induction that for every possible height $h \geq 0$, the smallest possible AVL tree with that height has $f_{h+3} - 1$ nodes. We are using f_n to refer to the nth Fibonacci number as defined in class.