Assignment 14, Math 240, Fall 2005

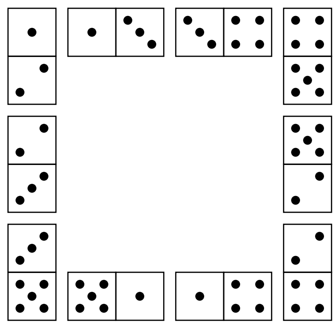
Due: 2:45pm, November 21. Value: 30 pts.

Based on November 15 material (§8.5)

Problem A. The set of *double-n dominoes* contains all pairs $\{i, j\}$ where $i \neq j$. (This removes the doubles, such as $(1, 1)$, which aren't interesting for our purposes.) For example, the set of double-5 dominoes includes ten dominoes:

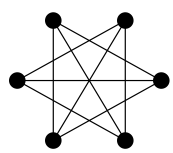
$$\{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{2,5\}, \{3,4\}, \{3,5\}, \{4,5\}.$$

Notice that these dominoes can be arranged in a circle where each domino end touches one other domino end with the same number.



For which n can you arrange a double-n domino set in a circle? Prove your answer.

Problem B. A *Hamiltonian circuit* is a circuit that visits each vertex of a graph exactly once. Identify a Hamiltonian circuit for $\overline{C_6}$, the complement of the 6-cycle.



Problem C. Prove that for every integer $n \geq 5$, $\overline{C_n}$ has a Hamiltonian circuit. Your proof should work by demonstrating the paths; it should *not* use theorems that imply the existence of Hamiltonian circuits. (The textbook presents a couple of such theorems without proving them.) Hint: Consider the case of odd n first.

Based on November 17 material (§8.7)

Problem D. Demonstrate that $\overline{C_6}$ is planar.

Problem E. Since C_6 is also planar, it is an example of a graph for which both it and its complement are planar. Show an example of a graph G with eight vertices for both G and $\overline{G}$ are planar.

Problem F. Prove that for every graph G with at least 11 vertices, either G or $\overline{G}$ is nonplanar. Hint: We saw in class that a planar graph with m edges and n vertices must have $m \leq 3n - 6$.