Take-Home Test 1, Math 240, Fall 2005

Due: 2:45pm, September 13. Value: 50 pts.

Instructions: In contrast to assignments, you must not discuss this take-home test with others, and you may not use electronic resources. The only written resources you may use are your textbook and class notes that you have written yourself. Be careful not to place your test where others might read your solutions. You may discuss the take-home test with me, although I won't promise to be helpful.

You will have the opportunity to complete a "rewrite," due the class day following when I distribute your initial grades. You will receive a three-point penalty on the rewrite, and your overall grade will be the higher of your initial grade and your rewrite grade. Your first graded version must be submitted with your rewrite.

Because of the rewrite policy, you should be careful not to discuss the test with others or access related resources even *after* you have submitted your initial version.

Problem 1. Prove that for all integers a, b, and d where

$$\left. \frac{d}{gcd(d,a)} \,\right|\, b\,,$$

it is true that $d \mid ab$. You should make use of Theorem 1 on page 154 whenever possible; you should also make use of the fact that whenever $m \mid n$ for two integers m and n, then $mk \mid nk$ for any integer k.

Problem 2. Define $F(n)$ to be the sum of all the divisors of n, excepting n itself. For example, $F(12)$ is $1 + 2 + 3 + 4 + 6 = 16$. Argue that for all integers s and t where s and t are relatively prime, $F(st)$ is $F(s) \cdot F(t)$.

Problem 3. Consider the set of solutions to the equation $ax^2 = b$ for all integers a and b where $a \neq 0$; this set includes $\sqrt{2}$ (from $a = 1$, $b = 2$), $1/2$ (from $a = 4$, $b = 1$), and 0 (from $a = 1$, $b = 0$). Prove that this set is countable.